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NOISE PREDICTIONS FOR OPTOELECTRONIC OSCILLATORS USING DIFFERENT MODELS.

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ABSTRACT

In this paper an optoelectronic oscillator (OEO) is analized with two different approaches. The two approaches lead to two models which allow prediction of the oscillator phase noise on the basis of the noise of its components. The first method, called Leeson's method, applies control system theory to an oscillator described by functional blocks. The second, called Van der Pol's method, uses the time domain equation of the oscillator, which is linearized about a equilibrium under the assumption af slow variations of the state variables.

In both cases the predicted noise is compared with the measured noise of a real OEO.

1. INTRODUCTION

The two models derived in this paper allow the analysis of the various noise contributions in the new generation of microwave oscillators called opto-electronic oscillators (OEO) [2], [3].

In 1966 Leeson presented a simple linear model which can be used to predict the effect of amplifier phase noise on oscillator phase noise.[1] This model is based on heuristic considerations of the basic oscillator scheme: a frequency selective element and an amplifier in a closed loop configuration.

The application of control system theory in the frequency domain allows us to derive results that contain those stated by Leeson. Furthermore in this case the number of elements is not limited to an amplifier and a resonator, and the system structure can be more complicated.

The OEO has also been analyzed using a different approach, with the application of the Van der Pol method, which linearizes the time domain equations of the oscillator with the hypothesis of *slow variations* of signal's phase and amplitude about the oscillation condition.

Both models are linear. Otherwise, it would be impossible to describe the system through the transfer function of its functional block (Leeson's method), and it would also be impossible to perform the Fourier transform of the time domain equations (Van der Pol's method).

Therefore these two models are useful to analyze the noise processes associated with the oscillation signal without investigating the non linear dynamics involved in the oscillator.

Noise prediction from both models are compared with the experimental results obtained at NIST on an OEO operating at 10.6 GHz [3].

2. THE NOISELESS OPTOELECTRONIC OSCILLATOR

A key element in all OEOs is the optical fiber that provides the long delay which will determine the oscillation frequency. A second selective element chooses among the large number of possible oscillator modes created by the fiber. The oscillator also contains a laser, a detector, and an electro-optic amplitude modulator (EOM).

After optical detection, the RF signal is amplified and fed back to the modulator, thus closing the loop. The basic scheme is shown in Fig. 1.

In our case the EOM is a Mach-Zehnder type modulator, which has a cosine-shaped transmittance versus drive voltage

$$P_{out}(t) = P_{in}\,\gamma\left[1+\varepsilon\cos\left(\pi\frac{V_{bias}+e_{out}(t-\tau_d)}{V_\pi}\right)\right], \quad (1)$$

where P_{in} and P_{out} are the optical power incident on the modulator and detected at the end of the fiber. The modulator's parameters are γ, a factor related to the insertion loss; ε, a factor related to the extinction ratio; and V_π, the voltage that is required to move from a maximum to a minimum of the optical power transmittance. The bias point of the modulator is chosen to be one half of V_π.

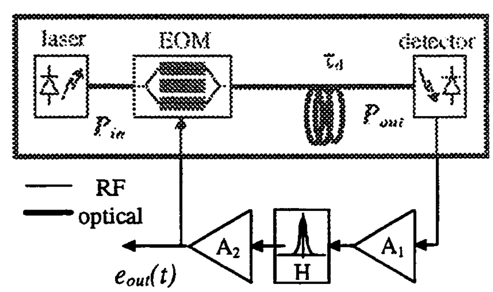

Figure 1. Basic scheme of an opto-electronic oscillator.

We assume that the signal around the loop will have the form

$$e_{out}(t) = V_0 \cos(\omega_0 t), \tag{2}$$

where $\omega_0 = 2\pi\nu_0$ is the oscillator angular frequency.

After expansion of the cosine in Eq. (1), Eq. (1) can be written as

$$P_{out}(t) = P_{in}\gamma\left[1 - 2\varepsilon\sum_{n=0}^{\infty} J_{2n+1}\left(\tfrac{V_0\pi}{V_\pi}\right)\cos((2n+1)\omega_0(t-\tau_d))\right]. \tag{3}$$

Harmonic distortion in the EOM is neglected because the filter in Fig. 1 selects only the fundamental frequency. In general this element is a bandpass filter; in our case is a microwave resonant cavity with a Lorentzian transfer function

$$H(\omega) = \frac{H_0}{1 + j2Q_L\dfrac{\omega-\omega_R}{\omega_R}} \cong H_0 e^{-j2Q_L\frac{\omega-\omega_R}{\omega_R}}, \tag{4}$$

where Q_L is the loaded quality factor and ω_R is the resonance frequency of the cavity. The exponential approximation in Eq. (4) is valid when

$$\omega - \omega_R << \frac{\omega_R}{2Q_L}; \tag{5}$$

that is, the cavity needs to be close to resonance at the oscillator frequency. The bandwidths of all the remaining elements in the oscillator, such as amplifiers, the detector, and the modulator, are considered to be wide enough that they will not affect the dynamics of the system. In particular the amplifiers' transfer functions will be

$$A_1(\omega) = A_{10}\, e^{-j\Phi_1} \quad \text{and} \quad A_2(\omega) = A_{20}\, e^{-j\Phi_2}. \tag{6}$$

The detected signal is simply $\rho P_{out}(t)$, where ρ is the detector responsivity. If we follow the rest of the signal path from the detector back to the modulator, we obtain

$$\begin{aligned} &e_{out}(t) = A_{20}H_0A_{10}\rho P_{out}(t-\tau_{RF}), \\ &\text{with} \quad \tau_{RF} = \frac{\Phi_1+\Phi_2}{\omega_0} + 2Q_L\frac{\omega_0-\omega_R}{\omega_0\omega_R}, \end{aligned} \tag{7}$$

which, using Eqs.(2) and (3), can be written as

$$\begin{aligned} &V_0\cos(\omega_0 t) = -2A_{20}H_0A_{10}\rho\varepsilon\gamma J_1\left(\tfrac{V_0\pi}{V_\pi}\right)\cos(\omega_0(t-\tau)), \\ &\text{with} \quad \tau = \tau_d + \tau_{RF}. \end{aligned} \tag{8}$$

It is now possible to write the oscillation condition as

$$\begin{cases} V_0 = 2A_{20}H_0A_{10}\rho\gamma P_{in}\,\varepsilon J_1\left(\tfrac{V_0\pi}{V_\pi}\right), \\ \omega_0\tau_d + (\Phi_1+\Phi_2) + 2Q_L\dfrac{\omega_0-\omega_R}{\omega_R} = (2K+1)\pi. \end{cases} \tag{9}$$

In absence of the resonator, the oscillation frequency is determined by the fiber alone:

$$\omega_F = \frac{2(K+1)\pi - (\Phi_1+\Phi_2)}{\tau_d}. \tag{10}$$

The fiber has a free spectral range of $1/\tau_d$. The effective oscillation frequency ω_0 depends on both cavity and fiber and its expression can be derived from Eq. (9):

$$\omega_0 = \frac{\omega_F\tau_d + 2Q_L}{\omega_R\tau_d + 2Q_L}\omega_R. \tag{11}$$

The detuning between the effective oscillation frequency and that determined by the fiber depends on both the initial detuning between the two selective elements and the two parameters Q_L and τ_d:

$$\omega_0 - \omega_F = \frac{2Q_L}{\omega_R\tau_d + 2Q_L}(\omega_R - \omega_F). \tag{12}$$

These are the working conditions for an oscillator with noiseless components; the derivation of the transfer functions for the noise spectra in the oscillator differs in the two approaches that will be addressed as the *Leeson's method* (application of control system theory) and the *Van der Pol's method* (time domain analysis).

3. LEESON'S METHOD

3.1 The noise transfer functions (NTF)

This method applies, in the frequency domain, control system theory to the transfer functions of the elements which describe the oscillator.

In particular, we represent the ensemble of the laser, the EOM, the fiber, and the detector as a whole "black box", called *optical link*. This optical link has $e_{out}(t)$ as input signal and the detected $e_d(t)$ as output signal. Recalling Eq. (3), we write the transfer function of the optical link in the frequency domain as

$$L(\omega) = \frac{E_d(\omega)}{E_{out}(\omega)} = -L_0 e^{-j\omega\tau_d}. \tag{13}$$

The system described by the "black boxes" defined above is shown in Fig. 2, with all the transfer functions involved.

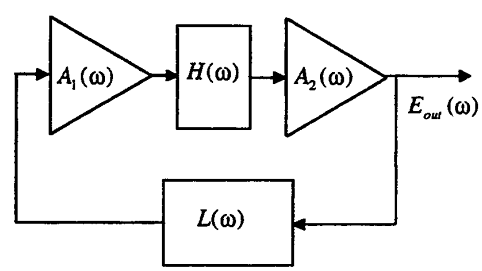

Figure 2. Block scheme of the optoelectronic oscillator.

To investigate the noise in the system, we make two assumptions. First, using the vectorial representation of a noisy signal shown in Fig. 3, we consider separately the amplitude and phase contribution of the noise component of the signal, and therefore

$$
\begin{aligned}
S_\varphi(f) \quad &\Leftrightarrow \quad (\Delta\Phi(f))^2 = \left(\frac{V_n(f)}{V_0\sqrt{2}}\right)^2, \\
S_\alpha(f) \quad &\Leftrightarrow \quad \left(\frac{\Delta V(f)}{V_0}\right)^2 = \left(\frac{V_n(f)}{V_0\sqrt{2}}\right)^2,
\end{aligned} \tag{14}
$$

where both thepower spectral densities of amplitude (α) and phase (φ) are represented. Second, it is possible to represent the noise of a device as an input equivalent noise which is transferred to the output by a noiseless transfer function.

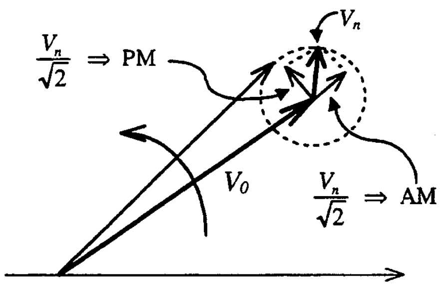

Figure 3. Vectorial representation of a noisy signal.

As a consequence of these two assumptions, the block diagram of the system with noise sources appears as shown in Fig. 4.

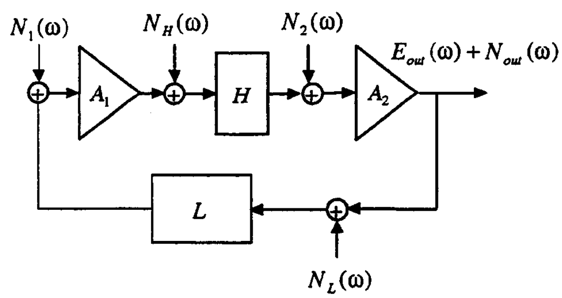

Figure 4. Block diagram of the noisy OEO.

These noise spectra can represent phase noise or amplitude noise as shown in Eq. (15),

$$
N_i(\omega) = \begin{cases} V_i \cdot \Delta\Phi(\omega-\omega_0) & \text{(PM)}, \\ \Delta V(\omega-\omega_0) & \text{(AM)}, \end{cases} \tag{15}
$$

where $i = 1, H, 2, L,$ and V_i is the signal at the respective summing junction.

The complete set of transfer functions for all noise contributions deduced from Fig. 4 is

$$
\begin{aligned}
&\text{1}^{\text{st}}\text{ amplifier} \quad && N_{out} = \frac{A_1 H A_2}{1 - L A_1 H A_2} N_1, \\
&\text{cavity} && N_{out} = \frac{H A_2}{1 - L A_1 H A_2} N_H, \\
&\text{2}^{\text{nd}}\text{ amplifier} && N_{out} = \frac{A_2}{1 - L A_1 H A_2} N_2, \\
&\text{optical link} && N_{out} = \frac{1}{1 - L A_1 H A_2} N_L.
\end{aligned} \tag{16}
$$

If we substitute the explicit expression for phase noise or amplitude noise shown in Eq. (15), the transfer functions for all the noise contributions and for both kind of noise spectra appear to be the same and can be expressed as

$$
NTF(\omega) = \frac{1}{1 - L A_1 H A_2} = \left[1 + L_0 A_{10} H_0 A_{20} e^{-j\left(\Phi_1+\Phi_2+\omega\tau_d+2Q_L\frac{\omega-\omega_R}{\omega_R}\right)}\right]^{-1}. \tag{17}
$$

Because the analysis takes place around the oscillation frequency, we can write

$$
\omega = \omega_0 + \Omega, \tag{18}
$$

where $\Omega/2\pi = f$ is the frequency offset from the carrier (Fourier frequency). Through application of Eq. (9), (10) and (12), we obtain

$$
\begin{aligned}
&NTF(\Omega) = \frac{1}{1 - e^{-jF(\Omega)}}, \\
&\text{with} \quad F(\Omega) = \Omega\left(\tau_d + \frac{2Q_L}{\omega_R}\right) + \\
&\qquad 2Q_L \frac{\omega_R\tau_d(\omega_0-\omega_F) - 2Q_L(\omega_R-\omega_0)}{\omega_R(\omega_R\tau_d + 2Q_L)},
\end{aligned} \tag{19}
$$

and therefore the transfer function of the power spectral densities appears as

$$
|NTF(\Omega)|^2 = \frac{1}{[1-\cos(F(\Omega))]^2 + \sin^2(F(\Omega))}. \tag{20}
$$

If there is no detuning between the oscillation frequency set by the fiber and the resonance frequency of the cavity, the Eq. (19) is simplified, and we can add some consideration. In fact if the Fourier frequency is small enough, the noise transfer function can be approximated by

$$
|NTF(\Omega)|^2 \approx \frac{1}{F^2(\Omega)} = \frac{\omega_0^2}{\Omega^2(\omega_0\tau_d + 2Q_L)^2}, \tag{21}
$$

$$
\text{with} \quad \Omega \ll \frac{1}{\tau_d} \quad \text{and} \quad \Omega \ll \frac{\omega_0}{2Q_L}.
$$

This is a simple result, identical to that found by Leeson in his paper.

3.2 Prediction on the experimental device.

In order to apply this model and calculate the total expected noise spectrum of an OEO, we have to know the equivalent input noise of all components in the system. It is also possible simply to calculate the effect of a single component on the spectral quality of the signal produced by the oscillator.

In our case the phase noise of the amplifiers is known from direct measurement [3], and its effect calculated using this model, is given by

$$S_\varphi(f)\Big|_{osc}^{pred} = \frac{S_\varphi(f)\Big|_{amp}}{\left[1-\cos\left(\Omega\tau_d + \frac{2Q_L\Omega}{\omega_0}\right)\right]^2 + \sin^2\left(\Omega\tau_d + \frac{2Q_L\Omega}{\omega_0}\right)} \qquad (24)$$

and is compared with the measured oscillator phase noise [3] in Fig. 5.

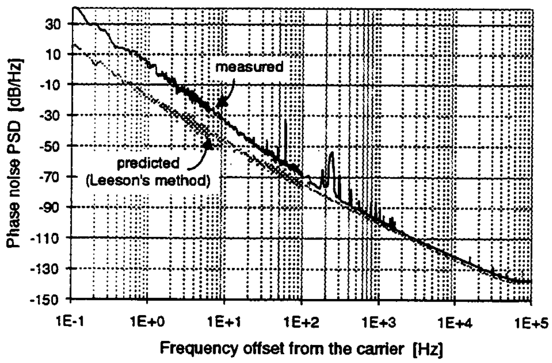

Figure 5. PSD of the measured oscillator noise and predicted amplifier phase noise contribution according to the model achieved with the Leeson's method.

The discrepancy between the two curves for Fourier frequencies below 100 Hz is due to the presence of excess noise on the oscillator signal, caused mainly by environmental factors like thermal fluctuation and vibrations.

4. VAN DER POL'S METHOD

4.1 The time domain analysis

This analysis is done mainly in the time domain and is based on the assumption that the oscillator signal can be written as

$$e_{out}(t) = V(t)\cos\left(\omega_0 t + \varphi(t)\right), \qquad (22)$$

with both amplitude and phase varying *slowly* in time. The starting point is the oscillation condition expressed by Eq. (8) and rewritten as

$$V(t+\tau)\cos(\omega_0 t + \omega_0\tau + \varphi(t+\tau)) = -2GP_{in}\,\varepsilon\gamma \cdot J_1\left(\tfrac{V(t)\pi}{V_\pi}\right)\cos\left(\omega_0 t + \varphi(t)\right), \qquad (23)$$

where

$$G = A_2 H A_1 \rho,\quad \tau = \tau_d + \frac{\phi}{\omega_0} + 2Q_L\frac{\omega_0 - \omega_R}{\omega_0\omega_R},\quad \phi = \phi_1 + \phi_2. \qquad (24)$$

Under the assumption of *slow variations*, we can write

$$V(t+\tau) = V(t) + \dot V(t)\tau + \ddot V(t)\tau^2,\quad \varphi(t+\tau) = \varphi(t) + \dot\varphi(t)\tau + \ddot\varphi(t)\tau^2, \qquad (25)$$

with $\tau >> 2\pi/\omega_0$.

Using Eqs. (23) and (25), we can write the differential equation which describes the amplitude and phase fluctuations of the oscillation signal:

$$\left(V + \dot V\tau + \ddot V\tau^2\right)\cos\left(\omega_0 t + \omega_0\tau + \varphi + \dot\varphi\tau + \ddot\varphi\tau^2\right) = -2GP_{in}\,\varepsilon\gamma\, J_1\left(\tfrac{V\pi}{V_\pi}\right)\cos\left(\omega_0 t + \varphi\right), \qquad (26)$$

where are $V(t)$ and $\varphi(t)$ the independent variables.

The stationary condition is represented by the derivatives of the state variables equal to 0. Under these assumptions, we recover the oscillation condition already calculated in Eq. (9):

$$\begin{cases} V_0 = 2G_0 P_{in}\,\gamma\varepsilon\, J_1\left(\frac{V_0\pi}{V_\pi}\right), \\ \omega_0\tau_0 = (2K+1)\pi \qquad \text{with} \qquad \tau_0 = \tau(\omega_0). \end{cases} \qquad (27)$$

The parameters of the system which affect its equilibrium are: the loop gain G, the optical power P_{in}, the fiber delay τ_d, the phase delay introduced by the amplifiers ϕ, and the resonance frequency of the cavity ω_R.

If Eq. (26) is linearized around the stationary condition with respect to both the independent variables and the parameters, it is possible to calculate its Fourier transform. The linearized equations appears as

$$\begin{cases} \left(\frac{V_0\pi}{V_\pi}\frac{J_0\left(\frac{V_0\pi}{V_\pi}\right)}{J_1\left(\frac{V_0\pi}{V_\pi}\right)} - 2\right)\cdot\delta V(t) - \tau_0\cdot\delta\dot V(t) - \tau_0^2\cdot\delta\ddot V(t) \\ \qquad + \frac{V_0}{G_0}\cdot\delta G(t) + \frac{V_0}{P_{in}}\cdot\delta P_{in}(t) = 0, \\ \tau_0\cdot\delta\dot\varphi(t) + \tau_0^2\cdot\delta\ddot\varphi(t) + \omega_0\cdot\delta\tau_d(t) + \delta\phi(t) \\ \qquad - 2Q_L\frac{\omega_0}{\omega_R^2}\cdot\delta\omega_R(t) = 0, \end{cases} \qquad (28)$$

where the Greek letter δ indicates difference with respect to the equilibrium value.

The application of the Fourier transform produces

$$\begin{cases} \dfrac{\Delta V(\Omega)}{V_0} = \dfrac{\left(\dfrac{\Delta G(\Omega)}{G_0}+\dfrac{\Delta P_{in}(\Omega)}{P_{in}}\right)}{2-\dfrac{V_0\pi}{V_\pi}\dfrac{J_0\left(\frac{V_0\pi}{V_\pi}\right)}{J_1\left(\frac{V_0\pi}{V_\pi}\right)}+j\Omega\tau_0-\Omega^2\tau_0^2}, \\ \Delta\varphi(\Omega) = \dfrac{\left(\omega_0\Delta\tau_d(\Omega)+\Delta\phi(\Omega)-2Q_L\dfrac{\omega_0}{\omega_R^2}\Delta\omega_R(\Omega)\right)}{\Omega\tau_0(\Omega\tau_0-j)}. \end{cases} \tag{29}$$

A direct consequence is the relationship between all noise power spectral densities involved in the system.

In particular, the phase noise of the oscillator, is

$$S_\varphi(\Omega) = \frac{\omega_0\tau_d\cdot S_{\Delta\tau_d/\tau_d}(\Omega)+S_\phi(\Omega)-2Q_L\dfrac{\omega_0}{\omega_R}\cdot S_{\Delta\omega_R/\omega_R}(\Omega)}{\Omega^2\tau_0^2(\Omega^2\tau_0^2+1)}. \tag{30}$$

Recalling Eq. (24), which contains the explicit form of τ, we can calculate its value at equilibrium and find the part of Eq. (30) relative to the amplifier phase noise (that is, $S_\phi(\Omega)$) which matches the results obtained with Leeson's method.

4.2 Prediction on the experimental device

As with previous method, we can use the results produced by this model to predict the effect of amplifier phase noise on the oscillator noise.

The transfer function that we are interested in is

$$S_\varphi(\Omega)\Big|_{osc}^{predicted} = \frac{S_\phi(\Omega)\big|_{amp}}{\Omega^2\tau_0^2(\Omega^2\tau_0^2+1)} \approx \frac{S_\phi(\Omega)\big|_{amp}}{\Omega^2\tau_d^2}, \tag{31}$$

where $S_\phi(\Omega)\big|_{amp}$ has been measured [3]. The measured noise of the OEO and the predicted amplifier noise contribution to it are compared in Fig. 6.

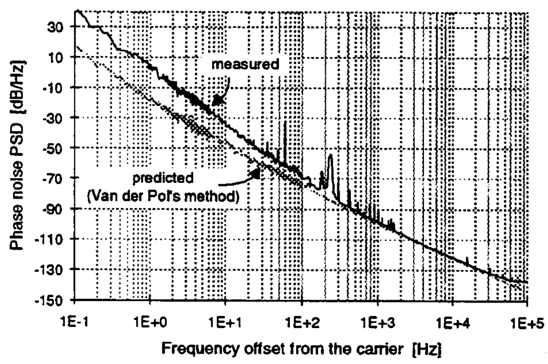

Figure 6. *PSD of the measured oscillator noise and predicted amplifier phase noise contribution according to the model achieved with the* Van der Pol's method.

The discrepancy between the two curves in the lowest frequency part of the spectrum is due to the presence of excess noise mainly caused by environmental factors as thermal variation and vibrations.

5. CONCLUSIONS

The two model described in this paper have different characteristics. The first model, calculated with control system theory (Leeson's method), is applicable when it is possible to describe the system in terms of black boxes whose associated noise is measurable without knowing the details of their operation.

The model calculated with Van der Pol's method permits us to identify the single contribution to the noise due to the fiber length variations or the cavity resonant frequency fluctuations. On the other hand, it is clearly less flexible and in fact requires the ability to write the system equations in the time domain and to verify all the assumption leading to the final expressions.

Although the two models presented belong to different analyses, their agreement with the experimental data and between them is good, as shown in Fig. 5 and Fig. 6.

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